

# TOWARDS APPLICATION OF A NEUTRON CROSS-SECTION UNCERTAINTY PROPAGATION CAPABILITY IN THE CRITICALITY SAFETY METHODOLOGY

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## ABSTRACT

At the Paul Scherrer Institute (PSI) the methodology for criticality safety evaluations (CSE) used for Swiss applications is focused, following common practice, on establishing an upper subcriticality limit (USL) for the effective neutron multiplication factor ( $k_{\text{eff}}$ ), based on a statistical evaluation of the obtained validation results in terms of the calculated-to-benchmark  $k_{\text{eff}}$  values. Recently, a new computational tool referred to as NUSS (Nuclear data Uncertainty Stochastic Sampling) was developed in order to propagate neutron-nuclear data (ND) uncertainties in calculations with Monte Carlo transport codes in combination with general-purpose ND libraries in ACE format. Thereby, this tool should now allow for a quantification of ND data uncertainties in CSE and to introduce these into the CSE criteria definition.

This paper concerns an application of the NUSS methodology for criticality validation studies. Calculated  $k_{\text{eff}}$  uncertainty estimations were obtained for the current PSI validation suite including in total 149 critical benchmark configurations for which in-house MCNP(X) models were developed, covering both uranium and mixed uranium-plutonium oxide fuels. The MCNPX-2.7.0 code was used together with the ENDF/B-VII.1 ND library while the ND variance-covariance matrixes (VCM) were taken from both SCALE-6.0 44-group libraries and also from ENDF/B-VII.1 files. Based on the newly obtained information on the uncertainties, a re-assessment of the lower tolerance bound (LTB) for  $k_{\text{eff}}$  using the total uncertainties as weighting factors is proposed as means for the present PSI CSE methodology upgrade and some possible directions of further methodology enhancements are discussed next.

## KEYWORDS

Criticality, uncertainties, cross-sections, Monte Carlo, ICSBEP benchmarks

## 1. INTRODUCTION

A conventional approach, required by the current standards (e.g. [1]) and employed in the state-of-the-art methodologies [2] for CSE in units of the effective neutron multiplication factor,  $k_{\text{eff}}$ , is based on the validation of the calculation tools, i.e. a code and a ND library, against representative benchmark experiments. A statistical analysis of the relations between the calculated and benchmark  $k_{\text{eff}}$  values

allows, in combination with an additional ‘minimum margin of subcriticality’ or an ‘administrative’ margin, derivation of a maximum allowable value for  $k_{\text{eff}}$ , or an Upper Subcriticality Limit (USL). Due to high construction and exploitation costs for spent nuclear fuel storage and transportation, there is an actual practical need to develop calculation methodologies which eventually may allow for more efficient design and operation of storage pools, transport casks, disposal canisters, etc. Obviously, this goal may be achieved if ever more confident qualification of CSE methodologies can be demonstrated based, e.g., on detailed elaboration of all foreseen uncertainty components in the calculation analysis, including those related to the  $k_{\text{eff}}$  uncertainties due to the ND.

Since Monte Carlo (MC) codes and point-wise ND libraries give the best possible accuracy for the criticality calculations, it is clearly an important task to provide a capability for ND uncertainties propagation with production Monte Carlo codes like MCNP and actually great progress has been achieved in this direction in the recent years [3,4,5], by development and implementation into MC codes of techniques for  $k_{\text{eff}}$  sensitivity coefficients calculations. Note that in the past the capabilities for ND uncertainties propagation with the MCNP(X) codes were principally limited by the Differential Operator perturbation option with known limitations [6,7].

Meanwhile, a tool NUSS [8] has been developed at PSI, which aims at ND uncertainties propagation with continuous-energy Monte Carlo calculations. Based on stochastic sampling approach, NUSS allows propagation of information on ND uncertainties characterized by group-wise VCMs through calculations with any Monte Carlo code employing the use of ACE-formatted libraries and for any output parameter. Use of the ACE-formatted libraries in combination with group-wise VCMs makes NUSS different from alternative stochastic-based tools TMC [9] and NUDUNA (see in [2]) and allows easy application of VCMs taken from different sources (and even ‘adjusted’ VCMs [2]) to facilitate verification studies. It is important to notice that one of the reasons for choosing the stochastic sampling approach instead of the deterministic based one, is that in addition to the ND uncertainties propagation, the same stochastic sampling method (i.e. within the same set of calculations) may be effectively applied for propagation of technological/design parameters uncertainties. No ready to use deterministic-based methodology with as versatile capabilities as the stochastic sampling method exists for such applications. Another potential benefit of the sampling-based methodology is that combination of the technological parameters and ND uncertainties can lead to non-linear effects which may be difficult to catch accurately enough with the deterministic-based sensitivity/uncertainty analysis techniques.

Thus the paper presents an application of NUSS/MCNPX-2.7.0 and ENDF/B-VII.1 ND library [10] in combination with SCALE-6.0 [11] and ENDF/B-VII.1 [10] VCM libraries for 149 benchmarks from ICSBEP Handbook [12] which constitutes currently the validation suite for the CSE methodology at PSI. A validation study of MCNPX-2.7.0 and ENDF/B-VII.1 has been performed at PSI recently [13]. For consistency reasons, in future use preference will be given to the ENDF/B-VII.1 VCM library and therefore the SCALE-6.0 VCMs are applied here mainly for comparison assessment and as additional means of the calculation procedures verification.

## **2. PSI CSE METHODOLOGY AND ITS VALIDATION APPROACH**

All ICSBEP benchmarks selected for the methodology validation involved non irradiated fuel and correspond to low-enriched compound thermal (LCT) and mixed compound thermal (MCT) categories, which are representative for LWR fuel configurations. Thus the benchmark suite is appropriate for validation of the methodology for fresh fuel condition only. There are 122  $\text{UO}_2$  and 27 MOX cases in the suite [14,15]. Inclusion of the MOX cases was important since this type of fuel is rather actively used at Swiss NPPs. One of the criteria applied at the selection of the CSE validation suite was a reasonable value of the benchmark uncertainties reported in benchmark specifications. Following the existing practices [16], the experimental benchmark uncertainties are treated in the PSI CSE methodology as weights for

$k_{eff}^{calc} / k_{eff}^{bench}$  processing. The underlying idea is that the  $k_{eff}^{calc} / k_{eff}^{bench}$  values obtained for the benchmarks with lower uncertainties (replicated by  $k_{eff}^{bench}$  uncertainties) shall receive higher weights in the overall statistical evaluations as compared to the cases with larger uncertainties. Based on the validation results obtained in the form of  $[k_{eff}^{calc} / k_{eff}^{bench}]$  sample, the weighted average  $\bar{k}$  and the weighted sample standard deviation<sup>1</sup>  $s$  can be calculated using the following formulas:

$$\sigma_i = \left( \frac{k_{eff}^{calc}}{k_{eff}^{bench}} \right)_i \sqrt{\left( \frac{\sigma^{bench}}{k_{eff}^{bench}} \right)_i^2 + \left( \frac{\sigma^{calc}}{k_{eff}^{calc}} \right)_i^2}; \quad w = \sum_{i=1}^N w_i; \quad w_i = \frac{1}{\sigma_i^2} \quad (1)$$

$$\bar{k} = \frac{1}{w} \sum_{i=1}^N w_i \left( \frac{k_{eff}^{calc}}{k_{eff}^{bench}} \right)_i; \quad s = \sqrt{\frac{1}{(N-1)} \sum_{i=1}^N w_i \left( \left( \frac{k_{eff}^{calc}}{k_{eff}^{bench}} \right)_i - \bar{k} \right)^2} \quad (2)$$

where  $(k_{eff})_i$  and  $\sigma_i$  are the eigenvalue and its standard deviation for a benchmark case  $i$ , the suffixes ‘bench’ and ‘calc’ refer to the experimental value and the calculated value, respectively.  $N$  is the number of benchmark cases. Here  $\sigma^{calc}$  corresponds to the calculation  $k_{eff}^{calc}$  uncertainty, which currently is only attributed to the MCNPX statistical uncertainty ( $=\sigma^{MC}$ ),  $\sigma^{bench}$  is the benchmark specification uncertainty reported in [12].

Assuming the probability distribution of the underlying  $\{k_{eff}^{calc} / k_{eff}^{bench}\}$  population is normal, it is possible to evaluate, e.g., a Lower Tolerance Bound [16,14] using the  $\bar{k}$  and  $s$  estimates (alternatively, distribution-free bounds may be considered when the hypothesis of normality is not appropriate):

$$K_{eff}^{LTB} = \bar{k} - k_1(\alpha, p, N) \cdot s, \text{ (conservatively, } \bar{k} \text{ may be replaced by } \textit{Min}(\bar{k}, 1.0)) \quad (3)$$

The coverage factor  $k_1$  is a function of the sample size  $N$ , a confidence level  $\alpha$  and a proportion  $p$  of the  $\{k_{eff}^{calc} / k_{eff}^{bench}\}$  population. Note that  $s$  can be taken as the sample standard deviation (Eq (2)), or, more conservatively, as a ‘pooled variance’, see [16].

The LTB value is further used to construct the final safety criterion USL [1,16] to be applied in practical evaluations,

$$k_{eff}^{calc} + \Delta k_{eff}^{calc} < USL = LTB|_{AOA} - \Delta k_{eff}^{AM} \quad (4)$$

Here AOA stands for Area of Applicability and it is defined based on the characteristics of the benchmarks from the validation set.

<sup>1</sup> Following the equation given in the “ICSBEP Guide to the Expression of Uncertainties” [13] for the best estimate of the variance under quadratic loss and for the assumption of the Gaussian distribution of the parameter, the factor (N-3) was used instead of (N-1) in Eq. (2) in the previous publications [14]-[16] for the sake of additional conservatism in the USL values derivation. However, hereafter the (N-1) factor will be used to derive the  $s$  values.

When an application case goes outside AOA with respect to any parameter, an additional safety margin ( $\Delta k_{eff}^{AOA}$ ) might be applied. As the  $\Delta k_{eff}^{calc}$  term, usually only  $\sigma^{MC}$  is considered and assuming that the Monte Carlo estimated  $k_{eff}$  is normally distributed,  $\Delta k_{eff} = 2\sigma^{MC}$  is typically applied to provide 95% confidence interval for the computed  $k_{eff}$ . Then  $(1-\alpha)$  and  $p$  values are also equal to 95% for consistency. As for the other uncertainties in the application case  $k_{eff}$  (design and ND related uncertainties), it is assumed that they all are covered by the LTB value with additional  $\Delta k_{eff}^{AM}$  margin.

### 3. NUSS APPLICATION TO ICSBEP BENCHMARKS

Utilizing the stochastic sampling principle, NUSS generates a number of ND libraries based on a reference one, with randomly sampled cross-sections and parameters, following the available VCMs and assuming currently that all data follow a normal multivariate distribution [14]. Once a sample set of data libraries is prepared, it can be applied straight forward to any application and for any output parameter with no need for modifications of the original code and input models. One drawback of the NUSS methodology is the relatively high computation burden, though in overall it requires minimum user's efforts. On the other hand, the methodology is free from the first-order approximation which is typical for sensitivity coefficients calculations. Important to note is that NUSS has undergone numerous verification studies including (but not limited by) comparison with such alternative tools as SCALE6.0/TSUNAMI and NRG Tallies code based TMC [9]. Special emphasis shall be given to the fact that the previously observed disagreement with TMC-based *nubar* uncertainty propagation discussed in [18] has been recently resolved and a very good agreement between NUSS and TMC results is reported finally in [19].

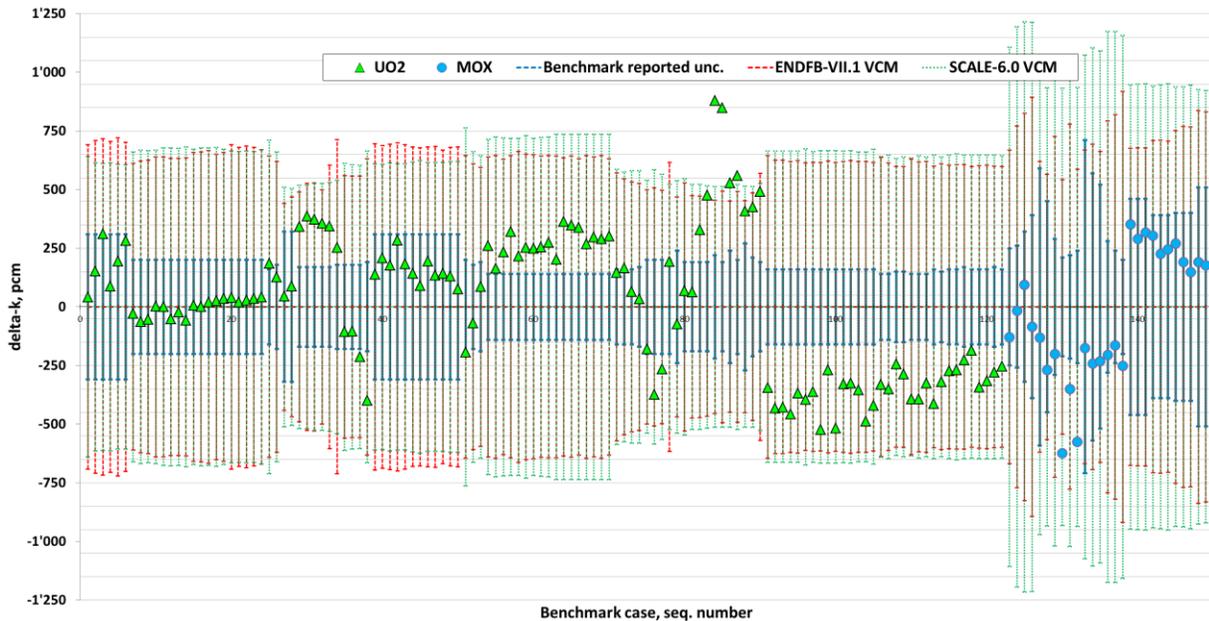
In case of SCALE-6.0 VCM libraries, the data is given in the 44-group structure, which is ready for use by the NUSS tool. In case of the ENDF/B-VII.1 library, the files MF-31, -32, -33, -35 need to be processed into group-wise format with NJOY code to be applicable with NUSS. The same 44-group structure was chosen here for generation of the VCM files using ENDF/B-VII.1 files. Previous studies did not reveal significant dependence of the resulting overall uncertainties on the group number unless it becomes too low [20,14]. Additional verification studies are currently on-going concerning the VCM group-wise collapsing. No indications that the use of the 44-group structure as implemented in SCALE libraries is not adequate to the uncertainties propagation in criticality calculations for LWR fuel were faced yet.

In the case of the ENDF/B-VII.1 library, 52 isotopes were varied with the NUSS methodology:  $^1\text{H}$ ,  $^9\text{Be}$ ,  $^{10,11}\text{B}$ ,  $^{16}\text{O}$ ,  $^{23}\text{Na}$ ,  $^{24-26}\text{Mg}$ ,  $^{27}\text{Al}$ ,  $^{28-30}\text{Si}$ ,  $^{41}\text{K}$ ,  $^{46-50}\text{Ti}$ ,  $^{50,52,53,55}\text{Cr}$ ,  $^{54,56,57}\text{Fe}$ ,  $^{59}\text{Co}$ ,  $^{58,60}\text{Ni}$ ,  $^{90-92,94,96}\text{Zr}$ ,  $^{92,94-98}\text{Mo}$ ,  $^{204,206-208}\text{Pb}$ ,  $^{234,235,238}\text{U}$ ,  $^{238-240,242}\text{Pu}$ ,  $^{241}\text{Am}$ . For non-actinides, cross sections such as (n,e), (n,inl), (n,2n) and (n,g) are considered, as defined in the MF-32/MF-33 covariance files (using the ENDF-6 nomenclature). For actinides, similar cross sections are varied with in addition (n,f), prompt nu-bar and the prompt fission neutron spectra, as defined in MF-31 and MF-35. In the case of the ENDF/B-VII.1 library, there is an important difference between using the prompt or total nu-bar covariance information. These two quantities were evaluated separately by the ND evaluators and are not consistent. The total nu-bar covariances lead to much higher uncertainties on  $k_{eff}$  compared to the prompt nu-bar covariance files. Therefore, until this problem is verified, the prompt nu-bar value is being used. The S(alpha,beta) tables and the probability tables (PT) are not sampled with NUSS yet. However, for the LCT type of systems the impact of the PT was found to be negligible [17], as may be confirmed by disabling the probability tables treatment in MCNP(X) calculations (relatively higher effects may be found in case of voided fuel rod lattices, but in such cases the reactivity of the LCT systems goes down significantly).

## 4. RESULTS

For each of the 149 benchmarks, 300 calculations were done with randomly sampled ACE-formatted libraries. MCNPX statistical uncertainties,  $\sigma^{MC}$ , of the individual runs were in the range of  $\sim 70$ - $90$  pcm. For verification of the numerical procedures, inspection was performed for: the ratios between the sample mean  $k_{eff}$  values and the  $k_{eff}$  values obtained with the nominal (reference, unperturbed) ACE libraries, the  $k_{eff}$  sample skewness and kurtosis. Finally the Kolmogorov-Smirnov normality test was performed for every sample set (300 samples for each of the 149 benchmarks). No test revealed any unexpected behavior of the results. The difference between the sample mean and the reference  $k_{eff}$  did not exceed  $\sim 2\sigma^{MC}$ .

A graphical representation of the obtained results is given below on Fig. 1. Both the experimental benchmark uncertainties and the cross-section related uncertainties values are shown as error bars at  $1\sigma$  level, with respect to the  $k_{eff}^{calc} / k_{eff}^{bench} - 1$  value equals to zero.



**Figure 1 Results obtained with ENDF/B-VII.1 ND library and VCM libraries from SCALE 6.0 and ENDF/B-VII.1.**

From the obtained results one can draw the following observations. At first, the obtained ND –related uncertainties are much larger the uncertainty estimates related to the benchmark specifications in most of the cases. Secondly, the uncertainties of the different cases within the same benchmark are usually quite similar and in overall the uncertainties within the two separate categories, UO<sub>2</sub> and MOX, are similar too. Uncertainties of the MOX cases are on average higher comparing to the ones of the UO<sub>2</sub> cases. More pronounced difference corresponds to the SCALE VCM case, while in the case of the ENDF/B-VII.1 VCM the difference is quite moderate. At the same time the benchmark uncertainties for the MOX cases are also generally higher comparing to the UO<sub>2</sub> cases. Regarding the MOX subset of benchmarks, one shall note that it is much smaller in size comparing to the UO<sub>2</sub> set.

Table I summarizes the results obtained only with ENDF/B-VII.1 VCM and shows that currently predicted uncertainties from the ND are about twice as high as the observed sample standard deviation

itself for the case of UO<sub>2</sub> fuel and even 3 times higher the standard deviation of only MOX cases. When analyzing the UO<sub>2</sub> and MOX cases individually, it can be also observed that the sample standard deviation of the MOX subset is smaller comparing the UO<sub>2</sub> case, regardless the fact that the estimated uncertainties of the  $k_{eff}^{calc} / k_{eff}^{bench}$  values (both from the benchmark side and from the calculation side, i.e. related to the ND) are higher in the case of MOX fuel.

**Table I. Validation benchmark subsets characteristics**

Benchmark cases	$N$	$\bar{\sigma}_{bench}^*$ , pcm	$\bar{\sigma}_{XS}$ , pcm	$1 - \bar{k}$ , pcm	$s$ , pcm
All	149	191	614	5	321
UO <sub>2</sub>	122	178	598	9	327
MOX	27	327	710	-56	217

\*)  $\bar{\sigma}_{bench}^2 = N / w$  (see Eq. (1)).

As concerns the SCALE 6.0 VCMs, it is known that they are being revised for SCALE 6.2 [21] and generally the uncertainties of the data relevant for LWR fuel criticality calculations have been changed in direction of reduction. Particularly, <sup>239</sup>Pu *nubar* and *chi* variances were significantly updated (mainly reduced) in the SCALE-6.2 version [21]. Thus it is expected that the uncertainty results, if obtained with SCALE 6.2 VCM data, must become closer to the ENDF/B-VII.1 results, especially for MOX cases. Since the choice for future reference calculations at PSI is presently ENDF/B-VII.1, only its results are present further in this paper.

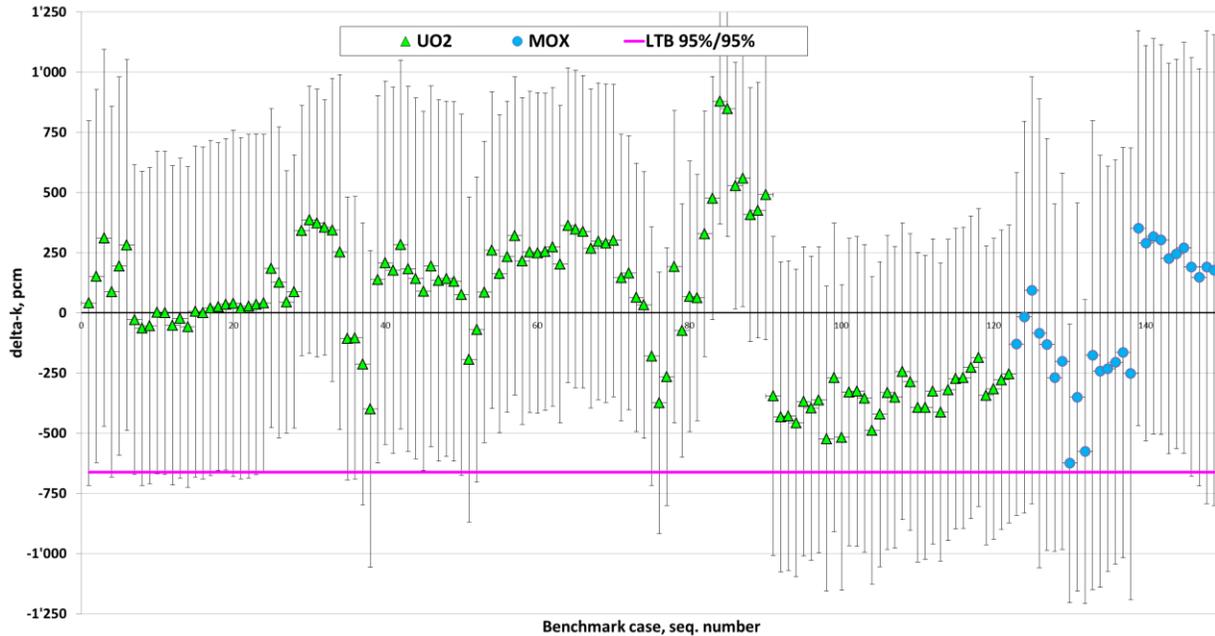
## 5. TOWARDS UPGRADING CURRENT PSI CSE METHODOLOGY FOR FRESH LWR FUEL

The new calculation capability of assessing ND related uncertainties brings two prospective ways for the CSE methodology upgrading. In the first one, the conventional approach expressed by Eq. (4) is retained, while the additional uncertainty data can serve as complementary information in the  $(k_{eff}^{calc} / k_{eff}^{bench})_i$  sample values weighting procedures, as discussed further in this section. Note that the conventional approach is mainly appropriate for the type of applications where good validation basis exists, like CSE for fresh LWR fuel.

When merging the UO<sub>2</sub> and MOX subsets into a single validation set (as is the case in the PSI reference methodology so far), a question may be posed, why the MOX cases having higher  $(k_{eff}^{calc} / k_{eff}^{bench})_i$  uncertainties have the same weights as more accurate UO<sub>2</sub> cases. The problem can hopefully be solved by re-assessment of the LTB value using the total uncertainties as weighting factors as given with Eq. (5). Here  $\sigma^{ND}$  stands for ND related uncertainty.

$$\sigma_i = \left( \frac{k_{eff}^{calc}}{k_{eff}^{bench}} \right)_i \sqrt{\left( \frac{\sigma^{bench}}{k_{eff}^{bench}} \right)_i^2 + \left( \frac{\sigma^{MC}}{k_{eff}^{calc}} \right)_i^2 + \left( \frac{\sigma^{ND}}{k_{eff}^{calc}} \right)_i^2}; \quad (5)$$

For illustration, Fig 2 shows the results together with the combined uncertainties (1 $\sigma$  level). As an example, the LTB value corresponding to 95%/95% proportion/confidence combination and using the sample standard deviation (2) in Eq. (3) is also shown on Fig. 2 for comparison.



**Figure 2 Combined uncertainties representation (ENDF/B-VII.1 ND & ENDF/B-VII.1 VCM libraries results).**

A slightly more penalizing LTB (and accordingly USL) value is obtained when Eq (5) is applied, as it is shown in Table II. It is then the updated LTB value which shall be selected for further applications (though the difference likely is really small, within  $3\sigma$  of MCNPX statistical uncertainty).

**Table II. Bounding LTB selection**

Parameter	$\bar{k}$	$s, pcm$	LTB 95%/95%
Reference results, Eq. (1,2)	0.99995	321	0.99395
Updated results, Eq. (2,5)	0.99951	328	<b>0.99339</b>

Obviously, the present PSI CSE methodology still has some room for further enhancements. An important aspect which eventually needs improvement is the question of similarity of the validation benchmarks to an application case. At present the only means provided within the PSI methodology to assess such similarity is verification that the main design characteristics and spectrum-related parameters of the application case fall within the related data ranges of the validation benchmarks. In case of any outliers, an additional penalty margin shall be applied to the USL value based on dedicated trend analysis [16,15]. However, no means are provided so far to take into account any quantitative merits of the similarity between an application case and the validation benchmarks at the USL value derivation (for example, a capability to assess similarity is implemented into SCALE package through the use of the sensitivity profiles and specific similarity indexes, e.g.  $c_k$  or others [3,22]; such integral indexes potentially may be used as weights in the USL definition [22], see also CEA and ORNL contributions in [2]).

In that sense the USL value can be an application case-specific. Works in this direction have been just started at PSI with present focus given to the further development and enhanced application of the NUSS-

RF (Random Balance Design and Fourier Amplitude Sensitivity Testing) [23] modification of the original NUSS version. The advantage of NUSS-RF is that it allows performance of the Global Sensitivity Analysis for correlated inputs (i.e. for ND).

It may be commented here in passing that another present drawback of the PSI methodology is the ignorance of the eventual correlations between the benchmark configurations from the validation suite. While the data on such correlations is still being prepared for the ICSBEP Handbook within the OECD/NEA WPNCs expert groups, another in-house tool, MTUQ (Manufacturing and Technological parameters Uncertainty Quantification) has been developed at PSI which potentially would be capable to contribute to the benchmark correlation assessment too [24].

Finally, concerning the ND uncertainties inclusion into the LTB derivation as weights, it is actually not evident that this approach is as meaningful as the inclusion of the benchmark specification uncertainties. Now the question may be formulated as follows: shall the related  $(k_{eff}^{calc} / k_{eff}^{bench})_i$  value receive lower weight in the USL evaluation, when the  $k_{eff}^{calc}$  uncertainty is relatively high? It seems that this approach cannot be fully justified. It can be the case that while the ND related uncertainties are relatively high for certain benchmarks, there are obvious reasons for that (e.g. nuclide composition and spectrum, resulting in higher uncertainties comparing to other benchmarks with different composition and spectrum combinations), and these benchmarks are actually those which are the most representative for an application case and thus assignment of lower weights to them must not be applied (e.g. for MOX type application case, the MOX benchmarks should not receive lower weights comparing to the UO<sub>2</sub> cases just because their ND related uncertainties are higher than those in the UO<sub>2</sub> case). At present, while the case specific, or “adaptive” USL value methodology is still being developed at PSI, it is proposed to follow a simple bounding solution, namely to select always the lower LTB value between the evaluations using Eq. (5) and the reference one, using Eq. (1).

## 6. DISCUSSION

In the future, currently missing sources of covariance information can be included, such as thermal scattering data for H in H<sub>2</sub>O (as presented in [1], where its impact ranges around 150 pcm for the LCT6 benchmark). Additionally, the contributions of the different isotopes to the total  $k_{eff}$  uncertainty can be obtained by changing only one isotope at a time (thus increasing the calculation time). As well, the Global Sensitivity Analysis will be performed with NUSS-RF. Finally, other ND library can be used, such as JENDL-4.0, or JEFF-3.2 (for the time being, JENDL-4.0 lacks covariances for fission products and JEFF-3.2 for the main actinides). As well, in addition to the  $k_{eff}^{calc}$  UQ, analysis of uncertainties of the spectrum-related parameters, which characterize the AOA of the CSE methodology [15] is currently planned. Needless to say that uncertainties in those calculation results (tallies) may be obtained using the stochastic sampling with NUSS basically at no additional costs comparing to the basic criticality calculations, but alternative calculations of sensitivity coefficients for such output parameters may become very complicated.

## 7. CONCLUSIONS

Explicit and detailed uncertainty analysis for an application system  $k_{eff}$  is not required in the current PSI CSE methodology, although all foreseen uncertainties are assumed to be reliably covered by USL for all normal and credible abnormal conditions. The USL value is defined once for the specific type of systems (e.g. LWR fresh fuel). The given approach is assumed to be conservative. Simple ways to reduce an excessive conservatism in the present approach are the increasing the size of the benchmark validation suite, as well as selection of higher quality benchmarks, which clearly are limited in practice by the

number of appropriate available benchmarks. At the same time, for certain ‘non-standard’ nuclear fuel configurations, which is frequently the case for research or experimental reactors (both types exist/existed in Switzerland), or for spent fuel (when burnup credit is considered) the available validation basis with experiments similar by their design/properties to the application case, may be very limited for statistically reliable evaluations. Appropriate analysis of the calculation uncertainties for an application case shall become a very valuable addition to the regular CSE procedures in such cases.

Therefore, an in-house tool for propagation of ND uncertainties in Monte Carlo calculations with ACE-formatted libraries has been developed at PSI recently to help for the CSE methodology enhancements. With this automatic tool at hand, it is now feasible to make uncertainty evaluations for any specific case of interest, as well as for entire validation base, which in case of PSI CSE methodology currently comprises 149 LCT and MCT benchmark cases in total.

Such study is presented in the given paper and it was based on application of the ENDF/B-VII.1 ND library in conjunction with SCALE-6.0 44-group and ENDF/B-VII.1 (processed to the same 44 groups) VCM libraries. Note that the SCALE-6.0 library is used here for verification purposes and main results’ evaluations are given for the reference computation route MCNPX-2.7.0/ ENDF/B-VII.1. Currently obtained results for  $k_{\text{eff}}$  calculation values demonstrate that with ENDF/B-VII.1 the ND related uncertainties for the entire PSI validation set are rather similar, in the range of  $\sim 0.6\%$  ( $\Delta k_{\text{eff}}$ ) for  $\text{UO}_2$  cases and  $\sim 0.7\%$  ( $\Delta k_{\text{eff}}$ ) for MOX cases.

Respective updates to the present PSI CSE methodology based on the new calculation capabilities have been discussed. Within the frame of conventional CSE approach, the total uncertainties of the  $k_{\text{eff}}^{\text{calc}} / k_{\text{eff}}^{\text{bench}}$  values may be considered as weights in the context of CSE USL evaluation. A more advanced, though more challenging approach would be to perform explicit uncertainties assessment for an application case. That is in line with the international trends in the field of criticality safety [2] and related methodology enhancements in this direction are currently under consideration at the Paul Scherrer Institute.

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