Uncertainty Propagation
with Fast Monte Carlo Techniques

D. Rochman°, S.C. van der Marck°, A.J. Koning°,
H. Sjöstrand* and W. Zwermann♣

° NRG, Petten, The Netherlands
♣ GRS Garching, Germany
* Uppsala University, Sweden
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1. 2008: TMC
   - Advantages
   - Drawbacks
2. Faster solutions
   - 2012: fast GRS Method
   - 2013: fast TMC
3. One example on full core
4. Conclusions

(Monte Carlo vs. Deterministic S/U...
Control of nuclear data (TALYS system) + processing (NJOY) + system simulation (MCNP/ERANOS/CASMO...)

For each random ENDF file, the benchmark calculation is performed with MCNP. At the end of the \( n \) calculations, \( n \) different \( k_{\text{eff}} \) values are obtained.

\[
\sigma_{\text{total}}^2 = \sigma_{\text{statistics}}^2 + \sigma_{\text{nuclear data}}^2
\]
Advantages of the TMC method

- many presentations at ND,
- computer time (not human time),
- Successfully applied (criticality, shielding, reactor, burn-up...)
- Most simple path (no additional processing, no covariance required),
- Other variants: AREVA (NUDUNA), GRS (XSUSA), CIEMAT (ACAB), PSI (NUSS), CNRS Grenoble..., based on covariance files,
- Many spin-offs (TENDL covariances, sensitivity, adjustment...)
- also applicable to fission yields, thermal scattering, pseudo-fission products, all isotopes (...just everything),
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But can TMC overtake (at least compete with) S/U methods?
Drawbacks of the TMC method

In TMC:

*If we can do a calculation once, we can also do it a 1000 times, each time with a varying data library.*
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There is a solution with Monte Carlo codes:

- fast GRS method,
- and fast TMC.
First presented in PHYSOR-2012 by W. Zwermann et al.. It takes advantage of conditional expectations:

If two output variables $k^{(1)}$ and $k^{(2)}$ are identically distributed and conditionally independent given the vector of nuclear data input then

$$\sigma_{\text{nuclear data}} = \sqrt{\text{cov}(k^{(1)}, k^{(2)})}$$
2012: fast GRS method

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$$

In practice:

1. perform $i = 1..500$ MCNP short calculations with random nuclear data and a fixed seed $s_1 \implies k^{(1)}_{\text{eff}}(i)$

2. repeat for $j = 1..500$, same random nuclear data but fixed seed $s_2 \implies k^{(2)}_{\text{eff}}(j)$

There is no necessity to have small $\sigma_{\text{statistics}}$ !! each run can be (very) short
fast GRS method

\[ \sigma_{\text{nucl data}} = \sqrt{\text{cov}(\tilde{k}_1, \tilde{k}_2)} = \sqrt{\text{corr}(\tilde{k}_1, \tilde{k}_2) \times \sigma_1 \sigma_2} \]

\[ = \sqrt{0.48 \times 0.0110 \times 0.0112} \]

\[ = 770 \text{ pcm} \]

\[ 2 \times 500 \text{ ”short” runs } \sim 2 \times \text{”long” run in time} \]
If a single calculation takes $m$ histories ($\sigma_{\text{stat}}$ small enough), then repeat it $n$ times with $m/n$ histories, random nuclear data and random seeds.

$$\sigma_{\text{total}}^2 = \sigma_{\text{statistics}}^2 + \sigma_{\text{nuclear data}}^2$$ still holds.
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still holds.

run 0  ENDF/B-VII.1  seed \( s_0 \)  \( m \) histories  T sec.  \( k \pm \sigma_{\text{stat}} \)
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<tr>
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<th>Histories</th>
<th>Time</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ENDF/B-VII.1</td>
<td>$s_0$</td>
<td>$m$</td>
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<td>$k \pm \sigma_{\text{stat}}$</td>
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<tr>
<td>1</td>
<td>Nuclear Data 1</td>
<td>$s_1$</td>
<td>$m/n$ hist.</td>
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<td>Nuclear data 2</td>
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| \vdots | \vdots | \vdots | \vdots | \vdots | \vdots |
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The fast methods

- as fast as S/U methods (1-2 × longer than 1 single calculation),
- tested on criticality & shielding benchmarks, burn-up (\(k_{\text{eff}}\) and inventory),
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- Example: the Martin-Hoogenboom benchmark

MCNP model: 241 fuel assemblies, with 264 fuel pins each

\[ 357 \times 357 \times 100 \text{ regions (1.26 \times 1.26 \times 3.66 cm}^3) : 12.7 \text{ million cells} \]
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Uncertainty on generated local pin power (tally f7) due to \(^{235}\text{U}, \(^{238}\text{U}, \(^{239}\text{Pu}\) and \(\text{H}\) in \(\text{H}_2\text{O}\) thermal scattering in each cell?
Fast TMC method

1 normal calculation without nuclear data uncertainty takes \( n = 2 \times 10^{11} \) histories
\( (\sigma_{\text{statistics}} = 0.25 \% \text{ at the center, 500 weeks on 1 cpu}) \)

\( \Rightarrow \) TMC: 500 random runs of \( n = 2 \times 10^{11} \) histories (500 weeks for each)

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**fast TMC and GRS methods:**

*If we can do a calculation once, we can also get nuclear data uncertainties in twice the time (or less).*